Buckling of Discretely Stringer-Stiffened Cylindrical Shells and Elastically Restrained Panels

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Theme

AN earlier parametric study of the buckling of discretely ring-stiffened cylindrical shells[1] revealed significant discreteness effects even for practical configurations with many rings. This motivated an extension of the study to stringer-stiffened shells and to elastically restrained panels.

The stiffeners are again considered as linear discontinuities represented by the Dirac delta function as in Refs. 1 and 2 and earlier analyses. The contributions of bending, stretching and torsional stiffnesses of stiffeners are taken into account as in other recent investigations[1-4]. The calculations are limited to uniform and equally spaced stiffeners, though the formulation permits nonuniformity in cross section and spacing. Only axial compression loading is studied since stringer-stiffening reinforces primarily against this loading.

The main purpose of the present paper is to apply the analysis for stringer-stiffened shells to a parametric study that will identify the combinations of shell and stringer parameters for which the discreteness effect is important. In most practical stringer-stiffened cylindrical shells outside a narrow range of geometries the discreteness effect is shown to be negligible.

Contents

Analysis: The stability equations for a cylindrical shell stiffened by discrete rings and stringers are given in Ref. 3. The equations are specialized to the case of no rings and uniform and equally spaced stringers. The displacement components are expanded into Fourier series in the circumferential direction

$$u = \cos m\beta x \sum_{t=1}^{\infty} A_t \sin t\phi \tag{1a}$$

$$v = \sin m\beta x \sum_{t=1}^{\infty} B_t \cos t\phi \tag{1b}$$

$$w = \sin m\beta x \sum_{t=1}^{\infty} C_t \sin t\phi \tag{1c}$$

Each set of terms of series [Eq. (1)] satisfies the stability equations of a shell with smeared stiffeners and each term fulfills the classical simple-support boundary condition

$$w=0,\ M_x=0,\ v=0,\ N_x=0 \quad \text{at } x=0,\ (L/R) \tag{2}$$

Substitution of the displacements into the stability equations yields three infinite sets of algebraic equations. The buckling load is found from the vanishing of the determinant of these equations. The buckling load parameter λ_d (where $\lambda = PR/\pi D$) is compared with that obtained for "smeared" stringers λ_s, and the difference in percent is defined as the discreteness effect

$$\Delta\lambda\% = 100(\lambda_s - \lambda_d)/\lambda_s \tag{3}$$

Results: The first set of results presents the effect of the discreteness of the stringers on the general instability of stringer-stiffened cylinders. Usually, a large number of stringers is needed to ensure that general instability is the failure mode and not local buckling. Thus, for most cases the magnitude of the discreteness effect is small. This part of the work identifies the combinations of shell and stringer parameters for which the effect is not negligible.

For a given shell and "smeared" stringer parameters there is a minimum number of stringers which is needed to ensure that general instability is the failure mode and not local buckling of the skin between stringers. (It is assumed that the stringers do not buckle locally as flanges. Such local stringer instability would occur only for very thin walled stringers which are avoided in practice.) The results presented in the following are for this minimum number of stringers and give, therefore, maximum discreteness effect for the combination of shell and "smeared"-stringer parameters. The "smeared" results were obtained with the analyses of Refs. 5 and 6.

The discrete results were obtained by using 20 terms in the Fourier expansion of Eq. (1). The first wave number is the "smeared" circumferential wave number t_o. The other 19 wave numbers were the lowest integers t_i such that either $(t_i + t_o)$ or $(t_i - t_o)$ is a multiple of the number of stringers n.‡

Figure 1 shows the magnitude of the discreteness effect, for fixed stringer parameters and various shell geometries. A very large value of the torsional rigidity parameter η_{t1} is chosen to emphasize the discreteness effect. For comparison, a few points have been calculated with $\eta_{t1} = 0$, shown as a dashed curve in Fig. 1. The Batdorf parameter Z is used to represent the shell geometry. The discreteness effect for external stringers, $(e_1/h) < 0$, is smaller than that for internal ones. Hence most of the numerical results presented here are for internal stringers, $(e_1/h) > 0$.

The buckling loads were calculated for a variety of stringer parameters for shells with $L/R = 0.5$, $R/h = 400$. The maximum effect found was less than 14%. The most important parameter is η_{t1}(defined as GI_{t1}/bD, where I_{t1} is the torsion constant of stringer cross section). The influence of the other parameters may be lumped in the parameter $(\lambda_s/\lambda_{uns})$, the ratio of the "smeared" buckling load of the stiffened shell to that of the corresponding unstiffened one. Figure 2 presents the discreteness effect as a function of a combined parameter $[\eta_{t1}/(\lambda_s/\lambda_{uns})]$. There is considerable scatter, but it is clear that for $\eta_{t1} = 0$ the discreteness effect is very small except for shells with extremely low Z (see Fig. 1).

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‡It is shown that only these wave numbers interact with t_o.

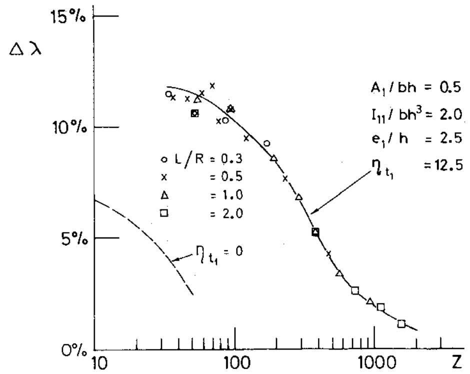

Fig. 1 Influence of shell geometry on discreteness effect.

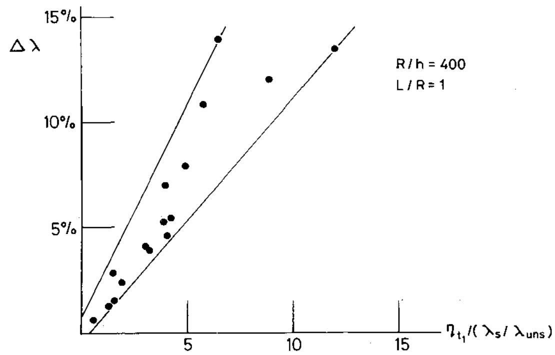

Fig. 2 Influence of stringer torsional stiffness on discreteness effect.

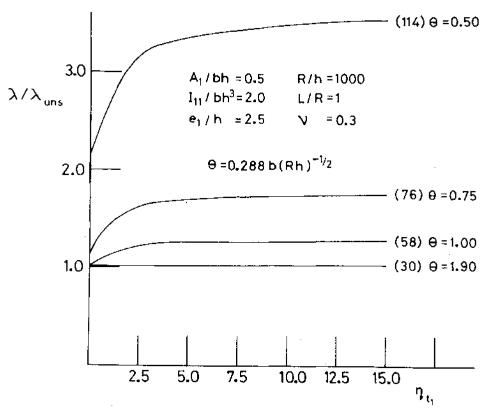

Fig. 3 Influence of stringer torsional stiffness on local buckling load.

In many cases of stringer-stiffened cylindrical shells and especially in light aircraft structures the local buckling load is lower than the general instability load, i.e., the panels between adjacent stringers buckle locally. The buckling load of the panels depends on the torsional rigidity of the stringers η_{t1}. The present theory furnishes a convenient tool for the calculation of this local buckling load. In the local buckling analysis the buckling load has to be calculated for either the wave number combination $n/2$, $3n/2$, $5n/2$ ... or n, $2n$, $3n$ ... where n is the number of stringers.

The dependence of the local buckling load on η_{t1} is presented in Fig. 3. It should be noted that for a small number of stiffeners, the torsional rigidity has almost no effect on the local buckling load.

In studying the effect of the elastic restraints due to the torsional stiffness of the adjacent stringers on the buckling of cylindrical panels, we should remember that wide panels buckle in the same mode and at the same critical stress as the corresponding complete unstiffened shell. Koiter[7] proposed a nondimensional parameter

$$\theta = \frac{[12(1-\nu^2)]^{1/4}}{2\pi}\,\frac{b}{(Rh)^{1/2}} = 0.288\,\frac{b}{(Rh)^{1/2}} \quad \text{for } \nu = 0.3$$

which defines the "narrowness' of the panel. For no rotational restraint (i.e., $\eta_{t1} = 0$), $\theta = 1$ indicates the point where the panel changes from narrow to wide. Figure 3 shows that even for large η_{t1} the narrow range does not extend beyond $\theta = 2$. Furthermore, the restrictions on θ are even more severe, if one considers initial post-buckling behavior[7] and for linear theory to apply the panel must comply with $\theta < 0.8$ even for large values of η_{t1}.[8] For wide panels, the usual large discrepancies between linear theory and experiment reappear.[9]

References

[1] Singer, J. and Haftka, R., "Buckling of Discretely Stiffened Cylindrical Shells," *Proceedings of the 10th Israeli Annual Conference on Aviation and Astronautics; Israel Journal of Technology,* Vol. 6, Feb. 1968, pp. 125-137; also TAE Rept. 67, Aug. 1967, Technion Research and Development Foundation, Haifa, Israel.

[2] Haftka, R. and Singer, J., "The Buckling of Discretely Stiffened Conical Shells," TAE Rept. 90, July 1968, Technion Research and Development Foundation, Haifa, Israel.

[3] Baruch, M., "Equilibrium and Stability Equations for Discretely Stiffened Shells," *Israel Journal of Technology,* Vol. 3, June 1965, pp. 138-146.

[4] Block, D.L., "Influence of Prebuckling Deformations, Ring Stiffeners and Load Eccentricity on the Buckling of Stiffened Cylinders," the AIAA/ASME 8th Structural Dynamics and Materials Conference, Palm Springs, Calif., March 1967.

[5] Singer, J., Baruch, M., and Harari, O., "On the Stability of Eccentrically Stiffened Cylindrical Shells under Axial Compression," *International Journal of Solids and Structures,* Vol. 3, July 1967, pp. 445-470.

[6] Weller, T., "Further Studies on the Effect of In-Plane Boundary Conditions on the Buckling of Stiffened Cylindrical Shells," TAE Rept. 120, Jan. 1974, Technion Research and Development Foundation, Haifa, Israel.

[7] Koiter, W.T., "Buckling and Post-Buckling Behavior of a Cylindrical Panel under Axial Compression," Report S. 476, National Luchtvaart-laboratorium, Amsterdam, *Reports & Transactions,* Vol. 20, 1956.

[8] Stephens, W.B., "Imperfection Sensitivity of Axially Compressed Stringer Reinforced Cylindrical Panels Under Internal Pressure," *AIAA Journal,* Vol. 9, Sept. 1971, pp. 1713-1718.

[9] Esslinger, M.E. and Geier, B.M., "Buckling and Postbuckling Behavior of Discretely Stiffened Thin-Walled Circular Cylinders," *Zeitschrift fuer Flugwissenschaft,* Vol. 18, July 1970, pp. 246-253.